Box 7-2   Ernst Abbe & Numerical Aperture

Following Joseph Lister's successful development of the achromatic lens doublet for microscope objectives in the late 1820s, microscope manufacture and design was put on a much more rational footing, and businesses consequently flourished both in Europe and the UK. However, the basis of image formation was still not understood fully, and microscope producers became skilled at making microscope objectives on an empirical basis, lacking any theoretical foundation: they followed manufacturing protocols that worked without understanding why.

Carl Friedrich Zeiss was no different. Starting in 1846, his business flourished, and within twenty years had made 1,000 microscopes. Zeiss insisted on high standards – he wanted his name to be synonymous with the very highest quality and performance - and it was not unknown for him personally to take a hammer to those instruments that failed the test. However, the empirical method was time-consuming, expensive, and success was not wholly guaranteed. Some competitors even said that their microscopes were "*not* like those made in Jena"!

In 1866, Zeiss asked Ernst Abbe, then 26 years old, to join him and place microscope manufacture on a rational footing. Abbe was a highly-gifted and energetic personality with outstanding qualifications. His father was an extremely hard-working spinner, yet the family lived in poverty. In later life Abbe worked very hard with Zeiss to ensure good employment conditions for his workforce, and the Carl Zeiss Stiftung (Foundation) continues today.

$$d = \frac{\lambda}{2n \sin\alpha}$$

By 1871 Abbe had completed his calculations for lens design, with the elucidation of the Abbe sine condition and a complete understanding of microscope image formation by diffraction. Now competitors would claim that their microscopes "followed the scientific principles established in Jena". In Abbe's own words:

"[objective manufacture] was based on a precise study of the materials used, the designs concerned are specified by computation to the last detail – every curvature, every thickness, every aperture of a lens, so that any groping around is excluded".

Abbe realised that the resolving power was not unlimited, but depended upon numerical aperture and wavelength. These terms were related in Abbe's famous formula: $d = \lambda/2n.\sin\alpha$. The term $n.\sin\alpha$ is the numerical aperture (NA), which is useful for comparing the resolving power of one objective with another. Abbe devised a simple means of measuring the NA of an objective using the Abbe Apertometer. A very high refractive index glass block ($n_D = 1.625$) was used to project an image of a scale into the back focal plane of the objective, from which the numerical aperture could be read directly. It works with both dry and immersion objectives.

This useful instrument is no longer made, but it is possible to make one yourself. A few years ago directions for doing so were published in the *Proceedings of the Royal Microscopical Society*.

When he died in 1905, two memorials were erected in Jena to Ernst Abbe. The larger, more central, one in Carl-Zeiß-Platz is an octagonal temple-like building. However, if you go north, via Johannisplatz and onto Fürstengraben, you will find the Abbe memorial opposite the Botanical Garden. The sphere appears to be made of two lenses – condenser and objective. Around the circumference, in between, you will see orders of diffracted rays, and the resolving power formula*.

* This is Jan Hinsch's suggestion; I am grateful to Peter Evennett for pointing it out to me.

References:

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